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Modelling Spatial Extreme Value with Copula approach and Aplication (Case Study: Extreme Rainfall in Ngawi)

Layla Fickri Amalia
Hospitality Study Program, Tourism Departement, Bali State Polytechnic, Indonesia
Email: fickriamalia@pnb.ac.id

Abstract

Extreme event is a short scale rare phenomenon, but it has a quite a serious impact on the various aspects of life. Studies on the prediction of extreme rainfall that occurred in the region is needed to minimize the adverse effects of global climate, so the farmers and stakeholders will have a good knowledge about the climate. It is especially extreme rainfall events so anticipation can be done early, than the production of rice plant can be maximized and the losses can be minimized. To fulfil these needs, it requires statistical methods that can explain the extreme rainfall. Extreme Value Theory is a statistical method to identify extreme events. For data rainfall, snow, river flow, or temperature are classified as spacial data being a multivariate data because it's observed in several locations, and therefore Spatial Extreme Value Method is developed. In the case of Spatial Extreme Value, the approach is often used Copula approach and max-stable process. Copula approach assumes that marginal distribution of extreme values follow a uniform distribution. Therefore, this study discusses the Spatial modelling of Extreme Value with Copula approach in one of East Java's rice production centres is Ngawi Regency. It is hoped that the method will improve the accuracy of predicting extreme events.

Keywords: *Spatial Extreme Value, Copula, Extreme Rainfall*

1. INTRODUCTION

Global climate change can cause extreme events, such as extreme rainfall, extreme air temperatures, and storm intensity. This global climate change occurs due to the increase in the average world temperature (Ainurrahmah et al., 2022). East Java is one of the provinces that is taken into account in contributing to rice production nationally. Around 17% of national rice production comes from East Java (BPS, 2021). Five districts in East Java that are among the largest rice suppliers are Jember, Bojonegoro, Lamongan, Banyuwangi and Ngawi regencies. Of the five districts, the largest producer of rice production is Ngawi Regency. In addition to the largest rice supplier, Ngawi Regency is also an area with high rainfall (Extreme Rainfall) during the rainy season, so it is very prone to flooding (Rahamayani et al, 2019)

Extreme rainfall is a weather condition that occurs when, the number of rainy days recorded exceeds the average price in the month in question at the station. When the greatest rain intensity in 1 (one) hour during a 24-hour period and intensity in 1 (one) day during a one-month period exceeds the average. If there is a wind speed of > 45 km / h and air temperature >35 ° C or < 15 ° C, and rainfall exceeds 100 mm / day (BMKG, 2018). Extreme rainfall is of particular concern, as the event causes losses in the agricultural sector. The excessive availability of rainwater results in flooding and submergence of agricultural areas, resulting in damaged rice fields and crop failure. Studies on estimating extreme rainfall that occurs in an area are needed to minimize the adverse effects of global climate change that often occur, so that farmers and *stakeholders* will have a good knowledge of the climate. Especially extreme rainfall events, so that early anticipation can be done, so that rice crop production can be maximized and losses withextreme events on one variable and often applied to stock data. For rainfall, snow, river discharge, and temperature data are included as spatial data which is multivariate data because it is observed in several locations, therefore *a spatial extreme value* method was developed. In the case of multivariate data, the approaches that are often used are the copula approach and *the max-stable* process (Yasin et al, 2019)

There are several methods to analyze extreme events with *spatial extreme value*, including the copula approach (Prayoga et al, 2020). (Cooley et al, 2007) examined spatial extreme precipitation in Colorado with a *Bayessian hierarchical* approach. In addition, there is the *Max Stable Process* (*MSP*) method developed by de Haan (1984) and developed by several other researchers such as Schlather (2002), Kabluchko, Schlather, and de Haan (2009).

Copula is one of the statistical methods that can describe the relationship between variables that is not too strict on the assumption of distribution. Copula is a function of two distribution relationships, each of which has a marginal function of distribution (Prayoga et al, 2020). Syahir (2011) applies copula in the field of climatology. Ratih (2012) looked at dependencies and modeled with Copula Regression for the case of modeling rice harvest area in Jember Regency. Anisa (2015) conducted a copula approach for the analysis of rainfall relations and *El-Nino Southern Oscillation* indicators in East Java rice production centers. Furthermore, Sari (2013) identified and suspected extreme rainfall at 15 rainfall stations in Indramayu Regency. The results showed that the approach with copula gave precise results for extreme observational data. Estimasi parameter copula can be done in various ways, including: *maximum pairwise likelihood estimation* (MPLE), Tau Kendall approach and Rho-Spearman approach. Studies on estimating copula parameters for extreme rainfall cases are still not widely discussed. Therefore, in this study, a study was conducted on parameter estimation in copula with *maximum pairwise likelihood estimation* (MPLE). This research continues four previous studies with case studies modeling extreme rainfall in Ngawi Regency, East Java. Ngawi Regency was chosen because it is one of the central districts of food crop production (rice) in East Java with a contribution of 647,264 tons. In addition, Ngawi is an area that is prone to flooding, so if there is continuous extreme rain, of course, it affects crop yields or rice productivity. For this reason, a study was conducted in the Ngawi area to determine the behavior patterns of extreme events. Therefore, this study discusses parameter estimation in *Spatial Extreme Value* modeling with a copula approach in one of the East Java rice production centers, namely Ngawi Regency.

2. METHODOLOGY

2.1 Extreme Value Theory (EVT)

EVT is a theory studying about the probability of extreme by focusing on the tail behaviour of a distribution. The heavy tail of the distribution is bigger than the probability of extreme value to appear and explains that there are two methods of defining extreme value, such as BM and Peak Over Threshold (POT) (Huzer et al, 2022). EVT involves a distribution for generalize extreme data into distribution of GEV. X~GEV(μ, α, ξ) has the form of Probability Density Function (PDF) that define in equation (1).

$$f(x;\mu,\sigma,\xi) = \begin{cases} \frac{1}{\sigma}\left\{1+\xi\left(\frac{x-\mu}{\sigma}\right)\right\}^{-\frac{1}{\xi}-1} exp\left\{-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}, \xi \neq 0 \\ \frac{1}{\sigma} exp\left\{-\frac{x-\mu}{\sigma}\right\} exp\left\{-exp\left(-\frac{x-\mu}{\sigma}\right)\right\}, \xi = 0 \end{cases} \quad (1)$$

x is variable of observation, μ is a parameter of location, σ is a parameter of scale with σ > 0, and ξ is a parameter of shape. Parameter ξ indicates the tail behaviour of GEV. Based on parameter ξ. GEV distribution follows Gumbel distribution, when ξ =0, follows Frechet distribution, when ξ > 0, and follows Reversed Weibull distribution when ξ < 0.

2.2 Block Maxima (BM)

BM is extreme value identification method based on the formation of blocks period. Data of observation are divided into certain blocks. Based on the formed blocks, it is chosen by the maximum value of observation of each block. The chosen maximum value of each block belongs to extreme sample. The Extreme value taken using BM method follows GEV distribution (Buzer et al, 2018)

2.3 Spatial Extreme Value (SEV)

There are many data of observation connected with natural happening, such as data from a happening in a small area in the larger area. Based on the data, it is possible that there is dependency between one spot to another spot in one area of happening. For example $M(j,t)$ is an extreme event data on the locations and the block periods *t*, in the spatial domain. Distribution for $M(j,t)$ is:$M(j,t) \sim GEV(\mu(j,t), \sigma(j,t), \xi(j,t))$ (2)

where$\hat{\mu}(j)$, $\hat{\sigma}(j)$, and $\hat{\xi}(j)$ is parameter location, scale and shape from GEV. (Hakim et al, 2016)

2.4 Copula

Copula can explore and characterize the structure of dependencies between random variables with marginal distribution functions. Copula transformation is done by the original data into a pseudo-observation followed by estimation using maximum pairwise likelihood estimation. Suppose that a sample size of n and dimension m, l = 1,2, ..., n; j = 1,2, ..., m. By using the extreme value distribution function F indicated by equation (1), it can be obtained pseudo-observation indicated [4] with the transformation equation as follows:

$$u_j = F_{X_j}(x_{ij}) \quad (3)$$

For observation of spatial extreme data can use Elliptical Copula and one of the family Elliptical Copula is the Gaussian Copula (Genest et al, 2010).

2.5 Gaussian Copula

CDF Gaussian Copula (Genest et al, 2010) following equation :

$$C(u_1, u_2 \ldots, u_m) = \Phi(\Phi_1^{-1}(u_1), \Phi_2^{-1}(u_2), \ldots, \Phi_m^{-1}(u_m); \rho) \quad (4)$$

with

Φ : CDF gaussian distribution
ρ : correlation function

2.6 Maximum Pairwise Likelihood Estimation (MPLE)

MPLE is parameter estimation method using the function of density pairwise or in pair of variables. MPLE method replaces the function $\left(\mathrm{L}(\beta)\right)$ in MLE with the function of pairwise likelihood

$$L_p(\beta)\ell_p(\hat{\beta}) = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} ln\left(f\left(u_{ji}, u_{ki}; \hat{\beta}\right)\right) \quad (5)$$

$f\left(u_{ji}, u_{ki}; \hat{\beta}\right)$ is density pairwise Gaussian Copula with parameter β and i=1, 2, ..., n (Hakim et al, 2016).

2.7 Rainfall

Rainfall is the altitude of rainwater a rain meter on the flat place, not to absorb, not to permit, and not to flow. Extreme rainfall is rainfall which has the intensity of >100 millimeter per day. Rainfall with intensity of > 50 millimeter per day constitutes heavy rainfall.(BMKG, 2018)

3. RESULT AND DISCUSSION

3.1 Modelling Spatial Extreme Value with Copula Approach

Calculation of return level needs a number of estimator namely $\hat{\mu}(j), \hat{\sigma}(j),$ and $\hat{\xi}(j)$ which values have been counted first through a Trend Surface Model. Trend Surface Model

$$\hat{\mu}(j) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}\, u(j) + \hat{\beta}_{\mu,2}\, v(j)$$
$$\hat{\sigma}(j) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}\, u(j) + \hat{\beta}_{\sigma,2}\, v(j)$$
$$\hat{\xi}(j) = \beta_{\xi,0} \quad (6)$$

In its calculation needs the values of parameter β_μ, β_σ and β_ξ which must be estimated first from Copula. Parameter Estimation β_μ, β_σ and β_ξ Gaussian Copula using MPLE. First, Arranging PDF Gaussian Copula-based on CDF Gaussian Copula. CDF Gaussian Copula as follows:

$$C(u_1, \ldots, u_m) = \Phi(\Phi^{-1}[u_1], \ldots, \Phi^{-1}[u_m]) \quad (7)$$

PDF Gaussian Copula can be written as

$$c(u_1, \ldots, u_m) = exp\left(\frac{1}{2}(\Phi^{-1}[u_1], \ldots, \Phi^{-1}[u_m])^T . \left(\rho(h)\right)^{-1} \cdot (\Phi^{-1}[u_1], \ldots, \Phi^{-1}[u_m])\right) . |\left(\rho(h)\right)|^{-0.5} \quad (8)$$

Second, Arranging the function of the *pairwise likelihood* of PDF Gaussian Copula.

$$L_p(\hat{\boldsymbol{\beta}}) = \prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m} f(u_{ji}, u_{ki}; \hat{\boldsymbol{\beta}})$$
$$= \prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m} \left(f_{x_j}(x_{ji}) \cdot f_{x_k}(x_{ki}) \cdot \right. \quad (9)$$
$$exp \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \left(\frac{1}{2}(\Phi^{-1}[u_1]\ \Phi^{-1}[u_2])^T . (\rho(h))^{-1} \cdot (\Phi^{-1}[u_1]\ \Phi^{-1}[u_2])\right)$$
$$\left. \cdot\ \rho(h)|^{-0.5}\right)$$

Then, Function of *pairwise likelihood* is carried out into the form of ln (10)

$$\ell_p(\boldsymbol{\beta}) = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left(f_{x_j}(x_{ji}) \cdot f_{x_k}(x_{ki})\right) \cdot \left(\frac{1}{2}\left[\Phi^{-1}(u_{ji})\ \Phi^{-1}(u_{ki})\right]^T \cdot \left(\rho(h)^{-1}\right) \cdot \left[\Phi^{-1}(u_{ji})\ \Phi^{-1}(u_{ki})\right]\right) \cdot \left(\text{-}0.5 \ln|\rho(h)|\right)$$

Based on this function, there haven't parameter $\boldsymbol{\beta}$ which will be estimated. Nevertheless, the parameter $\boldsymbol{\beta}$ is appeared by elaborating upon variable u with its function of transformation. Which follows the transformation equations on (4).

$$u_j = F_j(x_{ij}) = exp\left\{-\left[1 + \xi_j\left(\frac{x_{ij} - \mu_j}{\sigma_j}\right)\right]^{-\frac{1}{\xi_j}}\right\} \quad (11)$$

And

$$\mu(j) = \boldsymbol{d_j^T \beta_\mu}$$
$$\sigma(j) = \boldsymbol{d_j^T \beta}_\sigma$$
$$\xi(j) = \beta_\xi = \beta_{0,\xi} \quad \text{with} \quad \boldsymbol{d_j} = \begin{bmatrix} 1 \\ u(j) \\ v(j) \end{bmatrix} \quad \boldsymbol{\beta_\mu} = \begin{bmatrix} \beta_{\mu,0} \\ \beta_{\mu,1} \\ \beta_{\mu,2} \end{bmatrix} \quad \boldsymbol{\beta_\sigma} = \begin{bmatrix} \beta_{\sigma,0} \\ \beta_{\sigma,1} \\ \beta_{\sigma,2} \end{bmatrix}$$

The last MPLE estimation process is derivation of function *ln pairwise likelihood* toward $\beta_\mu, \beta_\sigma\ dan\ \beta_\xi$.

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta_\mu}} = \frac{\partial \left(\begin{array}{l} \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left(\left\{\frac{1}{\mathbf{d_j^T\beta_\sigma}}\left[1+\mathbf{d_j^T\beta_\xi}\left(\frac{x_{ji}-\mathbf{d_j^T\beta_\mu}}{\mathbf{d_j^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_j^T\beta_\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d_j^T\beta_\xi}\left(\frac{x_{ji}-\mathbf{d_j^T\beta_\mu}}{\mathbf{d_j^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_j^T\beta_\xi}}}\right\}\cdot\right. \\ \left.\left\{\frac{1}{\mathbf{d_k^T\beta_\sigma}}\left[1+\mathbf{d_k^T\beta_\xi}\left(\frac{x_{ki}-\mathbf{d_k^T\beta_\mu}}{\mathbf{d_k^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_k^T\beta_\xi}}}\right\}\exp\left\{-\left[1+\mathbf{d_k^T\beta_\xi}\left(\frac{x_{ki}-\mathbf{d_k^T\beta_\mu}}{\mathbf{d_k^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_k^T\beta_\xi}}}\right\}\right)\cdot \\ \left[\begin{array}{l} \frac{1}{2}\left(\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d_j^T\beta_\xi}\left(\frac{x_{ji}-\mathbf{d_j^T\beta_\mu}}{\mathbf{d_j^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_j^T\beta_\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d_k^T\beta_\xi}\left(\frac{x_{ki}-\mathbf{d_k^T\beta_\mu}}{\mathbf{d_k^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_k^T\beta_\xi}}}\right\}\right)\right)\right)^T\cdot \\ \left(\rho(h)^{-1}\right)\cdot\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d_j^T\beta_\xi}\left(\frac{x_{ji}-\mathbf{d_j^T\beta_\mu}}{\mathbf{d_j^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_j^T\beta_\xi}}}\right\}\right)\ \Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d_k^T\beta_\xi}\left(\frac{x_{ki}-\mathbf{d_k^T\beta_\mu}}{\mathbf{d_k^T\beta_\sigma}}\right)\right]^{-\frac{1}{\mathbf{d_k^T\beta_\xi}}}\right\}\right)\right) \end{array}\right]\cdot\left(\text{-}0.5\ln|\rho(h)|\right) \end{array} \right)}{\partial \boldsymbol{\beta_\mu}}$$

In the above derivatives to further facilitate the decrease can be exemplified as follows:

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\mu}} = \frac{\partial (a \cdot b - c)}{\partial \boldsymbol{\beta}_{\mu}} = 0$$
$$= \frac{\partial a}{\partial \boldsymbol{\beta}_{\mu}} b + a \frac{\partial b}{\partial \boldsymbol{\beta}_{\mu}} - \frac{\partial c}{\partial \boldsymbol{\beta}_{\mu}} = 0 \quad (12)$$

With

$$\frac{\partial a}{\partial \boldsymbol{\beta}_{\mu}} = \left[\left(\frac{1}{\left\{\frac{1}{d_j^T\beta_\sigma}\left[1+d_j^T\beta_\xi\left(\frac{x_{ji}-d_j^T\beta_\mu}{d_j^T\beta_\sigma}\right)\right]^{-\frac{1}{d_j^T\beta_\xi}}\right\}}\left\{\frac{1}{d_j^T\beta_\sigma}\left(-d_j^T\beta_\xi\right)\left[1+d_j^T\beta_\xi\left(\frac{x_{ji}-d_j^T\beta_\mu}{d_j^T\beta_\sigma}\right)\right]^{-\frac{1}{d_j^T\beta_\xi}-1}\right\}\cdot\right.\right. \quad (13)$$

$$\left.\left.\cdot\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi\cdot(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right)+\left(\left\{\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}\right\}\cdot\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi\cdot(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right)\right]+$$

$$\left[\left(\frac{1}{\left\{\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}}\left\{\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\left(-\mathbf{d}_k^T\boldsymbol{\beta}_\xi\right)\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}\right\}.\right.\right.$$

$$\left.\left.\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi\cdot(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right)+\left(\left\{\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}\right\}\cdot\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi\cdot(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right)\right]$$

$$\frac{\partial b}{\partial \boldsymbol{\beta}_\mu} = \frac{1}{2}\left[\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}.\right.\right.$$

$$\left.\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi\cdot(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]$$

$$\left[\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{d_k^T\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}\right).\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}.\right.$$

$$\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\Bigg]^T\cdot\left(\rho(h)^{-1}\right).$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}\right)\right)+$$

$$\left(\phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}\right)\right).\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}$$

$$.\left(\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{d_j^T\beta_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}-1}\right).\left(\frac{\mathbf{d}_j^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_j)}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\ \phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}\right)$$

$$.\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}.\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}-1}.\left(\frac{\mathbf{d}_k^T\boldsymbol{\beta}_\xi.(-\mathbf{d}_k)}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\Bigg)$$

$$\left(\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_j^T\boldsymbol{\beta}_\xi\left(\frac{x_{ji}-\mathbf{d}_j^T\boldsymbol{\beta}_\mu}{\mathbf{d}_j^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_j^T\boldsymbol{\beta}_\xi}}\right\}\right)\Phi^{-1}\left(\exp\left\{-\left[1+\mathbf{d}_k^T\boldsymbol{\beta}_\xi\left(\frac{x_{ki}-\mathbf{d}_k^T\boldsymbol{\beta}_\mu}{\mathbf{d}_k^T\boldsymbol{\beta}_\sigma}\right)\right]^{-\frac{1}{\mathbf{d}_k^T\boldsymbol{\beta}_\xi}}\right\}\right)\right)^T$$

$$\cdot\left(\rho(h)^{-1}\right)\cdot \tag{14}$$

$$\frac{\partial c}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}}=0 \tag{15}$$

$\frac{\partial l(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}_{\boldsymbol{\mu}}}$ can be found in appendix 1 (16)

The way of analogy the same can be obtained decline $\frac{\partial l(\hat{\boldsymbol{\beta}})}{\partial \boldsymbol{\beta}_{\boldsymbol{\sigma}}}$ and $\frac{\partial l(\hat{\boldsymbol{\beta}})}{\partial \beta_\xi}$.From the results, parameter estimation results is not close form so the parameter estimates should be continued using numerical iteration. Numerical iterations were used in this study is a Quasi-Newton BFGS. Numerical iterasion method, BFGS Quasi-Newton constitutes the improvement of Newton Iteration Method. The equation of iteration method BFGS Quasi-Newton is :

1. Determining initial value $\theta^{(0)}$which can be filled with matrix in size of m with all its member is zero.
2. Determining $\alpha^{(k)}$, Whereas $\alpha^{(k)}$constitutes the function for minimizing error.

$$\alpha^{(k)} = argmin\left[f\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}+\alpha^{(k)}S^{(k)}\right)\right]$$
$$=\frac{S(\theta^{(k)})}{S(\theta^{(k)})^T H(\theta^{(k)})S(\theta^{(k)})} \tag{17}$$

3. Determining matrix $H^{(k+1)}$

$$\mathrm{H}^{(k+1)}=\mathrm{H}^{(k)}+\left(1+\frac{\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^{\mathrm{T}}\mathrm{H}^{(k)}\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)}{\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}}\right)\frac{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)\mathrm{T}}}{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)\mathrm{T}}\Delta\mathrm{g}\underset{\sim}{\boldsymbol{\theta}}^{(k)}}-\frac{\mathrm{H}^{(k)}\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)\mathrm{T}}+\left(\mathrm{H}^{(k)}\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)\mathrm{T}}\right)^{\mathrm{T}}}{\Delta\mathrm{g}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)\mathrm{T}}}$$

With
$H^{(k)}$=I (matrix identity in size of m)

$$\Delta g(\underset{\sim}{\boldsymbol{\theta}}^{(k)}) = g(\underset{\sim}{\boldsymbol{\theta}}^{(k-1)}) - g(\underset{\sim}{\boldsymbol{\theta}}^{(k)})$$

4. Determining $g(\underset{\sim}{\boldsymbol{\theta}}^{(k)})$namely matrix which elements containing the first derivation of $\underset{\sim}{\boldsymbol{\theta}}^{(k)}$
5. Determining $S^{(k)} = -(H^{(k)})g(\underset{\sim}{\boldsymbol{\theta}}^{(k)})$.

Where $\mathbf{H}^{(k)} = \begin{bmatrix} \frac{\partial^2 \ell(\beta)}{\partial {\beta_\mu}^2} & \frac{\partial^2 \ell(\beta)}{\partial \beta_\mu \, \partial \beta_\sigma} & \frac{\partial^2 \ell(\beta)}{\partial \beta_\mu \, \partial \beta_\xi} \\ \frac{\partial^2 \ell(\beta)}{\partial \beta_\mu \, \partial \beta_\sigma} & \frac{\partial^2 \ell(\beta)}{\partial {\beta_\sigma}^2} & \frac{\partial^2 \ell(\beta)}{\partial \beta_\sigma \, \partial \beta_\xi} \\ \frac{\partial^2 \ell(\beta)}{\partial \beta_\mu \, \partial \beta_\xi} & \frac{\partial^2 \ell(\beta)}{\partial \beta_\sigma \, \partial \beta_\xi} & \frac{\partial^2 \ell(\beta)}{\partial {\beta_\sigma}^2} \end{bmatrix}$ and $\mathbf{g}(\underset{\sim}{\boldsymbol{\theta}}^{(k)}) = \begin{bmatrix} \frac{\partial \ell(\beta)}{\partial \beta_\mu} \\ \frac{\partial \ell(\beta)}{\partial \beta_\sigma} \\ \frac{\partial \ell(\beta)}{\partial \beta_\xi} \end{bmatrix}$ (18)

6. Doing numerical iteration by using equation

$$\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)} \qquad (19)$$

where :

$\underset{\sim}{\boldsymbol{\theta}}^{(k)}$ = Parameter estimate in iteration k

7. Calculating $\Delta(\underset{\sim}{\boldsymbol{\theta}}^{(k)}) = (\underset{\sim}{\boldsymbol{\theta}}^{(k-1)}) - (\underset{\sim}{\boldsymbol{\theta}}^{(k)})$
8. Back to the process number 2 up to the process number 7

Iteration is done until $\left\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \right\| \leq e$ with e is the very small number (Murea, 2005)

3.2 Appllications

Parameter estimation in sub-chapter 3.1 is applied in the daily rainfall data Ngawi case. Spatial extreme value parameter estimation with Copula approaches are calculated each parameter and used trend surface models as an example in equation (6). Longitude (u) and latitude (v) are a geographic variable that indicate the coordinates for a location, in this case, serves as an explanatory variable as found in the regression models. Best known model with AIC 8356.333. Parameter estimation results are incorporated into the model in order to obtain the best trend surface model equation as follows:

$$\hat{\mu}(j) = -455,090 \text{ -68,06 } v(j)$$
$$\hat{\sigma}(j) = 135{,}571\text{-}0{,}885\ u\ (j)$$
$$\hat{\xi}(j) = -0{,}1578$$

The location, scale, and shape parameter estimation for each location can be determined using the model equations and the best variables surface trend latitude (v) and longitude (u) at each observation location. Copula parameter estimation for each location Stations in Ngawi presented in Table 1 as follows:

Table 1. Parameter Estimation with Copula Approach

Stations	*Latitude*	*Longitude*	*Location* $(\hat{\mu})$	*Scale* $(\hat{\sigma})$	*Shape* $(\hat{\xi})$
Gemarang	-7.396	111.366	48.268	37.057	-0.158
Guyung	-7.506	111.410	55.686	37.017	-0.158
Karangjati	-7.461	111.613	52.624	36.838	-0.158
Kedungbendo	-7.387	111.543	47.588	36.900	-0.158
Kedunggalar	-7.408	111.312	49.085	37.103	-0.158
Kendal	-7.560	111.289	59.430	37.125	-0.158
Kricak	-7.394	111.344	48.132	37.075	-0.158
Mantingan	-7.386	111.150	47.519	37.248	-0.158
Mardisari	-7.428	111.406	50.446	37.021	-0.158

Papungan	-7.383	111.369	47.383	37.053	-0.158
Paron	-7.437	111.396	51.058	37.030	-0.158

Based on Tables 3 and 4. there are differences in the level of return generated value. In the Post Gemarang. return levels generated through GEV amounted to 134.603. Meanwhile. through the Copula method. the resulting level of return is equal to 136.142.

Table 2. Annual return level 5 years' period with GEV

Stations	Testing	Training	$\|error\|$(%)
Gemarang	95	134.603	41,688
Guyung	130	127.458	1,955
Karangjati	85	145.183	70,803
Kedungbendo	99	137.314	38,701
Kedunggalar	116	151.838	30,894
Kendal	156	142.518	2,668
Kricak	89	132.799	60,133
Mantingan	138	142.616	3,345
Mardisari	141	138.079	2,072
Papungan	99	148.070	49,566
Paron	190	137.606	27,576

Table 3. Annual return level 5 years' period with Copula method

Stations	Testing	Training
Gemarang	95	136.142
Guyung	130	143.466
Karangjati	85	139.980
Kedungbendo	99	135.090
Kedunggalar	116	137.070
Kendal	156	147.465
Kricak	89	136.050
Mantingan	138	135.847
Mardisari	141	138.236
Papungan	99	135.249
Paron	190	138.869

4. CONCLUSION

From the results, parameter estimation results is not close form, so the parameter estimation should be continued using numerical iteration. Numerical iteration is used in this study is a Quasi -Newton BFGS. Based on the information BMKG, the difference between the forecast with actual data 25% -30% is still considered good. The value of the parameter estimated may predicted that there are 5 stations predicted value and the actual value of error below 30%, among other things Guyung, Kendal, Mantingan, Mardisari and Paron. Six other stations have predicted value to the actual value of more than 30%.

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